Collateral Unchained: Rehypotecation networks, complexity and systemic effects

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Introduction

- The use of collateral is of increasing importance for the functioning of the global financial system. In volume terms, collateral use has become over the past decade on par with monetary aggregates like M2 (Singh 2011, Singh and Stella, 2012).
- More precisely, ***rehypothecation*** gives the receiver of collateral the right to reuse it and pledge it in another transaction. This practice is good for the receiver as she can increase her liquidity by putting the collateral at use, instead of keeping it aside.
- Rehypothecation has thus clear beneficial effects in terms of increasing liquidity in financial markets.

Introduction

- At the same time rehypothecation may increase systemic risk as several lenders are counting on the same underlying collateral as backup in case things go wrong (Singh 2012).
- This implies that balance sheets of financial institutions become more interlocked because of rehypothecation, which then becomes yet another channel of distress propagation.
- The recent literature on rehypothecation (e.g. Bottazzi et al., 2012, Andolfatto et al., 2015, Monnet and Narajabad 2012, Maurin 2014, Singh, 2012) does not consider the role for the network structure for liquidity creation and distress propagation.
- In this work we investigate
  1. under which conditions the network of secured loans across agents generates additional liquidity in the market
  2. Which network structures generate larger liquidity losses following small local shocks.

Outline

1. A model of rehypothecation networks
2. Collateral multiplier and network structure with fixed hoarding coefficients
3. Collateral multiplier and liquidity hoarding cascades under VaR.

A sneak preview of the main results

- Not every network architecture generates endogenous collateral. Network cycles are necessary for that
- The size of the collateral multiplier is affected by features like the direction of collateral flows, cycles' length, network density.
- Emergence of trade-off between liquidity and systemic risk.
- Core-periphery graphs generate high collateral multiplier with a much lower density than random graphs. At the same time they also generate larger reduction in collateral flows following shocks.

A model of rehypothecation networks

- We consider a set of N financial institutions ("banks" henceforth)
- Banks invest into external assets and they lend each other on the interbank market.
- We assume that all debt contracts in this economy are repo contracts, they are thus secured by collateral.
- To collect funds via repo contracts each bank i ($1 \leq i \leq N$) can pledge its proprietary collateral or re-pledge the collateral obtained via reverse-repos.

A model of rehypothecation networks

Main variables

- $A_i^{C^{out}}$: the total amount of collateral **flowing out of the box** of the bank *i* (i.e. the total amount of collateral that the bank *i* use to obtain for loans from other banks).
- $A_i^{C^{rm}}$: the total amount of (re-pledgeable) collateral **remaining inside the box** of the bank *i*.
- A_i^C: the total amount of (re-pledgeable) collateral **flowing into the box** of the bank *i*. A_i^C includes proprietary as well as non-proprietary assets "receiving" from other banks. Note that by definition $A_i^C = A_i^{C^{out}} + A_i^{C^{rm}}$.
- A_i^0: the value of the **proprietary** collateral of the bank *i*.
- θ_i: the fraction of non-hoarded (i.e. outgoing) collateral of bank *i*.
- B_i: borrowers' set of bank i, i.e. the banks that obtained funding from *i* via repos and thus provided collateral to *i*. B_i is the in-neighborhood of *i*.
- L_i: lenders' set of bank i, i.e. the banks that obtained collateral from *i* and thus provided funding to *i*. L_i is the out-neighborhood of *i*.

A model of rehypothecation networks

Main variables

- δ_i: for every bank i, let k_i^{out} be its outgoing degree in the rehypothecation network, then δ_i is defined as[1]

$$\begin{cases} \delta_i = 1, & \text{if } k_i^{out} > 0 \\ \delta_i = 0, & \text{if } k_i^{out} = 0 \end{cases}$$

  Note that, $k_i^{out} = 0 \Leftrightarrow L_i = \varnothing$.

- $a_{i \leftarrow j}$: each $a_{i \leftarrow j}$ captures the direction of collateral flow from the bank i to the bank j. For every pair of banks i and j we define

$$\begin{cases} a_{i \leftarrow j} = 1, & \text{if bank } j \text{ gives collateral to bank } i \\ a_{i \leftarrow j} = 0, & \text{otherwise} \end{cases}$$

- $s_{i \leftarrow j}$: is the share of j's outgoing collateral flowing into i.

The determination of total collateral

- We can write the following expression for the dynamics of $A_i^{C^{out}}$, i.e. the total amount collateral **flowing out of the box** of the bank *i*

$$A_i^{C^{out}} = A_i^{0^{out}} + (1-h)\delta_i\theta_i \sum_{j \in B_i} s_{i \leftarrow j} A_j^{C^{out}} \quad (1)$$

- Similarly, the total amount of re-pledgeable collateral **remaining inside the box** of the bank *i* is

$$A_i^{C^{rm}} = A_i^{0^{rm}} + (1-h)(1-\delta_i\theta_i) \sum_{j \in B_i} s_{i \leftarrow j} A_j^{C^{out}} \quad (2)$$

- Finally, we can write the following expression for the dynamics of A_i^C, the total re-pledgeable collateral **flowing into the box** of each bank *i*

$$A_i^C = A_i^{C^{out}} + A_i^{C^{rm}} = A_i^0 + (1-h) \sum_{j \in B_i} s_{i \leftarrow j} A_j^{C^{out}} \quad (3)$$

The determination of total collateral

- The shares $s_{i \leftarrow j}$ obey some constraints. If $L_j \neq \varnothing$ (i.e. $k_j^{out} > 0$), the total outgoing collateral (pledged or re-pledged) by bank j is equal to

$$\sum_{i \in L_j} s_{i \leftarrow j} A_j^{Cout} = A_j^{Cout}$$

- The above implies

$$\sum_{i \in L_j} s_{i \leftarrow j} = 1, \quad \forall j = 1, 2, ...N \tag{4}$$

- Furthermore, if we assume that weights are homogeneous across lenders of j, then the elements of the matrix $\mathcal{S} = \{s_{i \leftarrow j}\}_{NxN}$ read as

$$\begin{cases} s_{i \leftarrow j} = \frac{a_{i \leftarrow j}}{k_j^{out}}, \text{ if } k_j^{out} > 0 \\ s_{i \leftarrow j} = 0, \text{ otherwise} \end{cases} \tag{5}$$

The determination of total collateral

- We shall mainly focus on the outflowing collateral $A^{C^{out}}$, as it represents the contribution of each bank to the overall funding liquidity of the system.
- In matrix form Equation (1) reads

$$A^{C^{out}} = A^{0^{out}} + (1-h)\mathcal{M}A^{C^{out}} \tag{6}$$

- The solution to the above equation returns the vector of outflowing collateral $A^{C^{out}}$.

$$A^{C^{out}} = (\mathcal{I} - (1-h)\mathcal{M})^{-1}A^{0^{out}} = \mathcal{B}_1 A^{0^{out}} \tag{7}$$

with $\mathcal{I}$ is the identity matrix of size N, $\mathcal{B}_1 = (\mathcal{I} - (1-h)\mathcal{M})^{-1}$, and the elements $m_{i\leftarrow j}$ of the matrix $\mathcal{M}$ are defined as

$$\begin{cases} m_{i\leftarrow j} = \delta_i\theta_i s_{i\leftarrow j} = \frac{\delta_i\theta_i a_{i\leftarrow j}}{k_j^{out}}, \text{ if } k_j^{out} > 0 \\ m_{i\leftarrow j} = 0, \text{ if } k_j^{out} = 0 \end{cases}$$

Network Structure and Total Collateral

- We now turn to investigate how the rehypothecation network (captured by the matrix $\mathcal{M}$) affects the total collateral in the system.
- We shall focus on how the network affects two main variables
  - the aggregate amount of outgoing collateral, $S^{out} = \sum_{i=1}^{i=N} A_i^{C^{out}}$
  - The multiplier of the amount of outgoing collateral,m, defined as

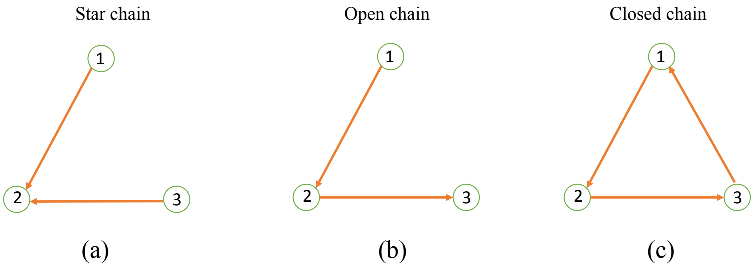
$$m = \frac{\sum_{i=1}^{i=N} A_i^{C^{out}}}{\sum_{i=1}^{i=N} A_i^{0^{out}}} = \frac{\sum_{i=1}^{i=N} \delta_i \theta_i A_i^C}{\sum_{i=1}^{i=N} \delta_i \theta_i A_i^0} \tag{8}$$

- We are in particular interested in identifying network structures that generate *endogenous collateral*, i.e. such that $m > 1$

Network Structure and Total Collateral

The role of the cycles

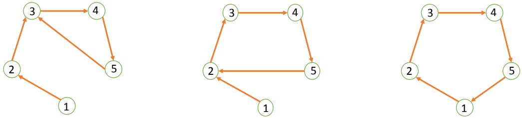
- Not every network structure generates endogenous collateral. For this to be true, the network must contain closed cycles.



Network Structure and Total Collateral

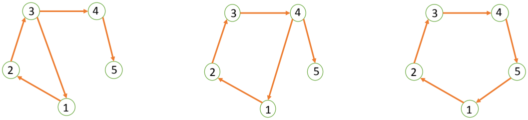
Direction of collateral and length of cycles

- Example 1. Same number of edges, different lengths of cycles, but same total amounts of outgoing collateral.



(a) (b) (c)

- Example 2. Same number of edges, different lengths of cycles; but total amounts of outgoing collateral increases with cycles' length.



(a) (b) (c)

Network structure and total collateral

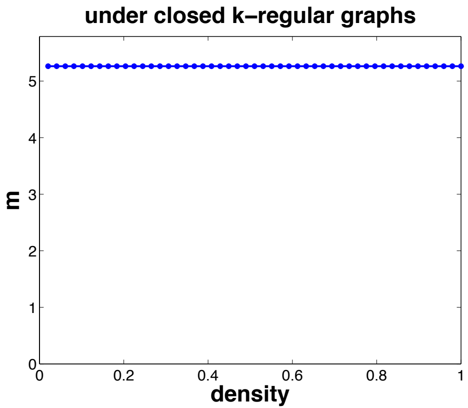


Figure: Collateral multiplier (m) as a function of density under closed k-regular graph Simulation is implemented with N=50, k=N-1, h=0.1, $1-\theta=0.1$, and $A^0=100$ for all banks.

Network structure and total collateral

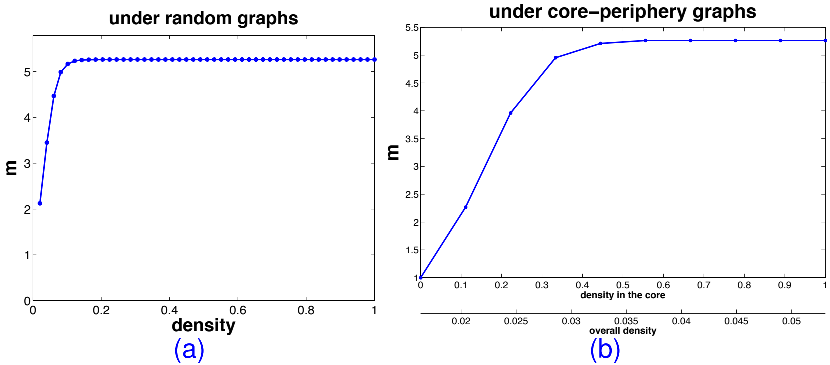


Figure: Collateral multiplier (m) as a function of density under random graph (a) and core-periphery graph (b). Simulation is implemented with N=50, h=0.1, $1-\theta = 0.1$, and $A^0 = 100$ for all banks.

- In the limit of density $\rightarrow$ 1 all the three structures generate the same multiplier $m^* = \frac{1}{1-(1-h)\theta}$

Value-at-Risk and collateral hoarding effects

- So far we have worked under the assumption that the shares on non hoarded collateral $\{\theta_i\}_{i=1}^N$ were homogeneous.
- We now remove this hypothesis and assume that banks that determine decide how much collateral to hoard on the basis of a Value at Risk criterion (VaR).
- This allows us to study cascades of hoarding following local shocks hitting fraction of nodes in different network structures, and more in general to study how the level of total collateral changes when the fraction of hoarded and non-hoarded collateral are heterogeneous across banks.

Value-at-Risk and collateral hoarding

- For every bank j let NL_j be its net liquidity position. Bank j has collateral A_j^C that can be used to get external funds with a haircut h.
- At the same time a fraction of this collateral θ_j is already pledged. The net liquidity position of the bank is thus:

$$NL_j = (1-h)(1-\theta_j)A_j^C(\theta_1, \theta_2, ..., \theta_N, G) - \epsilon_j \tag{9}$$

  where ϵ_j are payments within the periods, which are assumed to be a i.i.d. random variable (see also Gai, Haldane and Kapadia, 2011).
- The notation $A_j^C(\theta_1, \theta_2, ..., \theta_N, G)$ emphasizes the fact that the total collateral position of a bank depends - in general - on the fraction of hoarded collateral of all banks in the network G.
- In particular, the more borrowers of j hoard collateral, the lower is the value of collateral A_j^C, and thus the higher the need to hoard collateral for j.

Value-at-Risk and collateral hoarding

- Let us consider a realization of the liquidity shock that is sufficiently high to imply the default of bank j

$$\epsilon_j > (1-h)(1-\theta_j)A_j^C$$

- It follows that default is an event occurring with probability

$$\text{prob.}(NL_j \leq 0) = \text{prob.}(\epsilon_j > (1-h)(1-\theta_j)A_j^C)$$

- We assume that each bank j employs a Value-at-Risk (VaR) strategy to determine the fraction of collateral to hoard so that the above probability of default is not higher than a target $1 - c_j$.

$$\text{prob.}(\epsilon_j > (1-h)(1-\theta_j^*)A_j^C) \leq 1 - c_j \qquad (10)$$

- If we further assume that returns on assets held by j grow with asset size then each bank j will set its target $\theta_j = \theta_j^*$ so that

$$\text{prob.}(\epsilon_j > (1-h)(1-\theta_j^*)A_j^C) = 1 - c_j \qquad (11)$$

Value-at-Risk and collateral hoarding

- From the last equation it follows that θ_j depends on not only the uncertainty about ϵ_j (captured by the assumption about the distribution of ϵ_j but also on the haircut rate h as well as the value of the collateral A_j^C.
- The interdependence between θ_j and A_j^C implies that each bank will adjust its fraction of hoarded collateral depending on hoarding fraction set by other banks.
- If we assume that the shocks ϵ_j are either uniformly or normally distributed, the expression for the determination of the target θ_j^* becomes

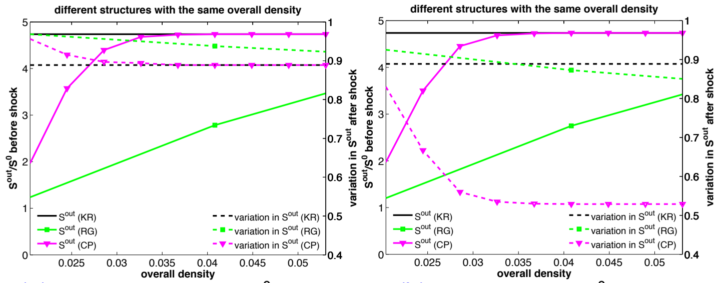
$$\theta_j^* = 1 - \frac{c_j^0}{(1-h)A_j^C}, \tag{12}$$

- Under the above distributional assumptions about ϵ_j the system determining the vector of collateral A^C and the vector of θ_j^* has a unique solution.

Collateral hoarding cascades

- We now study total collateral loss when - starting from a situation of homogeneous θ_j, a fraction of nodes experience a rise in uncertainty, captured by the parameter c_j^0 in Eq. 12.
- We study collateral loss under
  1. random and target attack
  2. different network densities and different network architectures (k-regular, random graph, core-periphery)

Collateral creation and collateral hoarding cascades in different networks



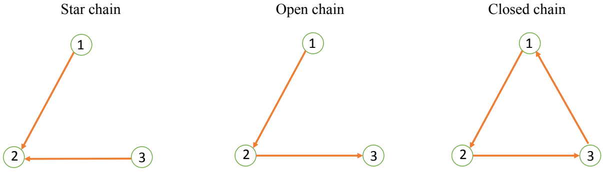
(a) random attacks at $\langle \tilde{c}_i^0 \rangle = 1.5$ (b) target attacks $\langle \tilde{c}_i^0 \rangle = 1.5$

Figure: Collateral creation and collateral variation under local uncertainty shocks. In each panel, the left y-axis shows $\frac{S^{out}}{S^0}$ (before shocks) and the right y-axis shows variation in S^{out} (after shocks) at $\langle \tilde{c}_i^0 \rangle = 1.5$. Different network structures are represented by different colors: for k-regular graphs in black, for random graphs in green, and for core-periphery graphs in magenta.

Conclusions

- We study how the structure of the network affects the level of total collateral in presence of rehypothecation.
- Closed chains (cycles) are necessary to generate endogenous collateral (i.e. a level of collateral higher than total proprietary collateral). Furthermore, the direction of collateral flows, network density and the length of cycles matter as well!
- Core-periphery networks generate larger multipliers with a much lower density than in other network structures (random graphs). At the same time they also also generate larger collateral losses in case of shocks.
- Take home message: to achieve both high liquidity and financial robustness the distribution of collateral flows should be as homogeneous as possible.

Network Structure and Total Collateral



(a) (b) (c)

- In the case of the star chain in the figure example (a)), B2 receives collaterals from B1 and B3 and no further pledge is made. We get
  - $A_1^C = (1 - \theta_1)A_1^0$,
  - $A_2^C = A_2^0 + (1 - h)(\theta_1 A_1^0 + \theta_3 A_3^0)$
  - $A_3^C = (1 - \theta_3)A_3^0$.
- It is easy to verify that $\sum_{i=1}^3 A_i^C = \sum_{i=1}^3 A_i^0 - h(\theta_1 A_1^0 + \theta_3 A_3^0) \leq \sum_{i=1}^3 A_i^0$.
- We obtain the same result for the open chain represented in case (b).

Network Structure and Total Collateral

- We now consider the third case where banks form a cycle (i.e. the closed chain in example (c)).
- If all banks take into account additional collaterals that they can receive from other banks (borrowers), then endogenous collateral is created (m>1)
- Denote $A_i^{C,t}$ the total collateral available for each bank i (including its proprietary as well as the re-pledged ones) after t rounds of re-using collaterals.

Network Structure and Total Collateral

- $\{A_i^{C,t}\}_i^3$ can be jointly determined by solving the following system of equations

$$\begin{pmatrix} A_1^{C,t+1} \\ A_2^{C,t+1} \\ A_3^{C,t+1} \end{pmatrix} = W \begin{pmatrix} A_1^{C,t} \\ A_2^{C,t} \\ A_3^{C,t} \end{pmatrix} + \begin{pmatrix} A_1^0 \\ A_2^0 \\ A_3^0 \end{pmatrix} \tag{13}$$

- where $W = \begin{bmatrix} 0 & 0 & (1-h)\theta_3 \\ (1-h)\theta_1 & 0 & 0 \\ 0 & (1-h)\theta_2 & 0 \end{bmatrix}$.
- Recursively we obtain the following dynamics

$$\begin{pmatrix} A_1^{C,t} \\ A_2^{C,t} \\ A_3^{C,t} \end{pmatrix} = [I + W^1 + W^2 + ... + W^t] \begin{pmatrix} A_1^0 \\ A_2^0 \\ A_3^0 \end{pmatrix}. \tag{14}$$

- Clearly, the endogenous creation of collateral in the system after t steps is given by $[W^1 + W^2 + ...W^t] \begin{pmatrix} A_1^0 \\ A_2^0 \\ A_3^0 \end{pmatrix}$.